Applied Scientific Research **59:** 111–125, 1998.

Direct Numerical Simulation of Turbulence in a Stably Stratified Fluid and Wave-Shear Interaction

MARTIN GALMICHE, OLIVIER THUAL and PHILIPPE BONNETON
Institut de Mécanique des Fluides de Toulouse, Allée du Professeur Camille Soula, 31400 Toulouse, France

Abstract. Turbulence decay in a strongly stratified medium is simulated by a direct pseudo-spectral code solving the three-dimensional equations of motion under the Boussinesq approximation. The results are compared to non-stratified simulations results. We focus on the production of mean shear energy observed in the stratified case. We then simulate the decay of stratified turbulence when affected by an initial horizontal mean flow and show that this mean flow is the major component remaining at large t. Next, we give some analytical elements on wave-shear interaction by using a simple refraction calculation with WKB hypothesis. This calculation is illustrated by simulating the interaction between one monochromatic internal wave and a vertical shear profile. We conclude that the existence of singularities in the mean shear production term in the presence of internal gravity waves may be one of the possible mechanisms involved within stratified turbulent shear flows.

Key words: stratified fluids, turbulence, mean flow, internal gravity waves, DNS.

1. Introduction

The evolution to anisotropy of turbulent flows under the influence of a vertical density gradient is still poorly understood. It induces the generation of an internal gravity wave field which is in strong interaction with the turbulent motions. This phenomenon was observed in the atmosphere and in the ocean's thermocline but its description remained wide open from a fundamental point of view.

In this short paper, some results from our current works on simulation of freely decaying sheared turbulence in a strongly stratified fluid are presented. The results are systematically compared to non-stratified analogous situations (see Section 2 for methodology).

Internal waves in the presence of a mean horizontal motion have already been studied from different points of view. Miles [12] gave a linear stability criterium for a uniformly stratified shear flow in terms of Richardson number. Bretherton [2] and McIntyre and Norton [10] studied the mean motion production by dissipation of internal waves. Booker and Bretherton [1] analytically studied critical layers and noticed that this non-linear phenomenon should lead to a high energy transfer to the mean flow. Koop [9] experimentally analyzed the wave field generated by an oscillating source in a stratified shear flow and observed the presence of critical layers.

We would like to highlight the importance of mean vertical shear within turbulent flows. Turbulent shear flows in stratified media have already been simulated by, for instance, Gerz and coworkers [5, 6] and Jacobitz et al. [8]. Experiments have been carried out in a thermally stratified wind tunnel by Piccirillo and Van Atta [13]. However, these numerical and experimental studies generally focus on the influence of a vertical mean shear on the stratified turbulence. In the present paper, we focus on the effect of stratified turbulence on the horizontal mean flow.

We first study (Section 3) the decay of an initially isotropic unsheared turbulent flow in the presence of a strong density gradient (the initial Froude number based on the integral scale is 0.1). Beyond the well-known wavy exchanges between poloidal and potential energies (see Section 2 for the definition of these energies), we focus on the time evolution of the mean shear component of the flow. The energy of this component, which starts with a very low value as the initial flow is isotropic, is observed to be not only slowly dissipated but also produced at the beginning of the decay. We then simulate (Section 4) a strongly stratified flow in the presence of an initial mean flow. This situation may be compared to the flow configuration in a turbulent wake. We observe a production of mean shear energy which in addition is slowly dissipated, so that this horizontal monodimensional flow is the major component remaining at large t (which is not the case in the non-stratified case). In Section 5, we propose a shear production mechanism involving the interaction between internal gravity waves and the mean flow itself. A WKB calculation leads to singularities in the shear production term due to wave refraction. As an illustration, we simulate the evolution of a flow for which the initial condition is the superposition of a mean flow and an isolated internal wave (Section 6). It is found that the wave rapidly transfers its energy to the mean flow.

2. Methodology

2.1. Equations of Motion

The dimensional equations of motion may be written in a Cartesian coordinate system $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ under the Boussinesq approximation as:

$$\nabla.\mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\frac{1}{\rho_0}\nabla p + \alpha g \theta \mathbf{e}_3 + \nu \nabla^2 \mathbf{u}, \tag{2}$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u}.\nabla)\theta = -Gw + K\nabla^2\theta, \tag{3}$$

where $\mathbf{u} = (u, v, w)$, θ and p are the velocity, temperature (or concentration) and pressure fields. g is the gravity, ν is the kinematic viscosity, K is the thermal diffusivity and α is the expansion coefficient. The density fluctuations are assumed to linearly depend of the temperature (or concentration) fluctuations: $(\rho-\rho_0)/\rho_0 = \alpha(\theta-\theta_0)$, where subscript 0 stands for a reference state. G is the mean temperature vertical gradient which one can express in function of the Brünt–Väisälä frequency N: $G = N^2/\alpha g$.

2.2. DIMENSIONLESS NUMBERS

The control parameters of the dynamical system are the Froude and Prandtl numbers, which describe the physical properties of the system. However, we choose to use a Reynolds number which includes a velocity scale of the initial conditions. Thus, the norm of the initial condition is numerically of order unity.

We will define our simulations in function of the three dimensionless numbers:

- $\mathrm{Fr} = U_{\mathrm{ref}}/Nl$, initial Froude number,
- $\mathrm{Re} = U_{\mathrm{ref}}l/\nu$, initial Reynolds number, and
- $\mathrm{Pr} = \nu/K$, Prandtl number.

In these definitions, U_{ref} is the velocity scale of the initial condition (root mean square velocity) and l is the initial integral scale of turbulence.

2.3. POLOIDAL-TOROIDAL-MEAN SHEAR DECOMPOSITION

We solve the Boussinesq equations with a pseudo-spectral direct simulation code [15] which uses the poloidal-toroidal-mean shear equations. One can write the velocity field as the sum of three non-divergent components:

$$\mathbf{u} = \mathbf{u}_{\mathrm{tor}} + \mathbf{u}_{\mathrm{pol}} + \mathbf{u}_{\mathrm{sh}}, \tag{4}$$

where $\mathbf{u}_{\mathrm{tor}}$ and $\mathbf{u}_{\mathrm{pol}}$ are called toroidal and poloidal velocities and $\mathbf{u}_{\mathrm{sh}}$ is the horizontally averaged velocity: $\mathbf{u}_{\mathrm{sh}}(z,t) = (U(z,t), V(z,t), 0)$. Components $\mathbf{u}_{\mathrm{tor}}(x,y,z,t)$ and $\mathbf{u}_{\mathrm{pol}}(x,y,z,t)$ are such that:

- the vertical component of $\mathbf{u}_{\mathrm{tor}}$ is zero, and
- the vertical vorticity of $\mathbf{u}_{\mathrm{pol}}$ is zero.

In highly stratified situations (i.e., when the Froude number is a small parameter), the poloidal component can be seen as the internal waves component [14].

Using the following variables:

- $\zeta(x,y,z,t)$, vertical vorticity,
- $w(x,y,z,t)$, vertical velocity, and
- $U(z,t)$ and $V(z,t)$, horizontal mean flow,

the poloidal, toroidal and shear energies can be written in the Fourier space as:

$$E_{\text{tor}} = \frac{1}{2} \sum_{\mathbf{k}} \hat{\zeta}(\mathbf{k}) \frac{1}{k_H^2} \hat{\zeta}^*(\mathbf{k}), \tag{5}$$

$$E_{\text{pol}} = \frac{1}{2} \sum_{\mathbf{k}} \hat{w}(\mathbf{k}) \frac{k^2}{k_H^2} \hat{w}^*(\mathbf{k}), \tag{6}$$

$$E_{\text{sh}} = \frac{1}{2} \sum_{\mathbf{k}} \left[\hat{U}(\mathbf{k}) \hat{U}^*(\mathbf{k}) + \hat{V}(\mathbf{k}) \hat{V}^*(\mathbf{k}) \right], \tag{7}$$

where $\hat{}$ stands for the Fourier Transform, * denotes complex conjugates, $\mathbf{k}$ is the wave vector, k its norm and k_H its horizontal component.

Averaging on the whole periodic box, the total energy may be expressed as:

$$E_{\text{tot}} = E_{\text{pol}} + E_{\text{tor}} + E_{\text{sh}} + E_{\text{pot}},$$

where E_{pot} is the potential energy:

$$E_{\text{pot}} = \frac{1}{2} \int_{R^3} \theta^2(\mathbf{x}) \, \mathrm{d}\mathbf{x}.$$

2.4. Initial Conditions

We first generate in the spectral space a random velocity field following a gaussian statistic and an energy spectrum given by:

$$\frac{E(k)}{E_0} = \frac{32}{3k_I} \sqrt{\frac{2}{\pi}} \left(\frac{k}{k_I} \right)^4 e^{-2(k/k_I)^2} ,$$

where $E(k)$ is the energy associated with the wave number k, E_0 is the integral of the spectrum and k_I is the injection wave number (peak in the spectrum, taken equal to 4.76). From this velocity field, we performed a short calculation without stratification in order to get a non-Gaussian velocity field where non-linear transfers are established (this simulation was stopped after 1.76 turnover timescales). The velocity field obtained is used as the initial condition for the simulation of the turbulence decay under the influence of a stable stratification. Then, the integral lengthscale l of turbulence is such that l/L_b is of order five where L_b is the size of the periodic cubic box. This value (which is necessarily limited by the resolution) insures that the most energetic structures of turbulence are correctly simulated. The initial density perturbation field is chosen equal to zero.

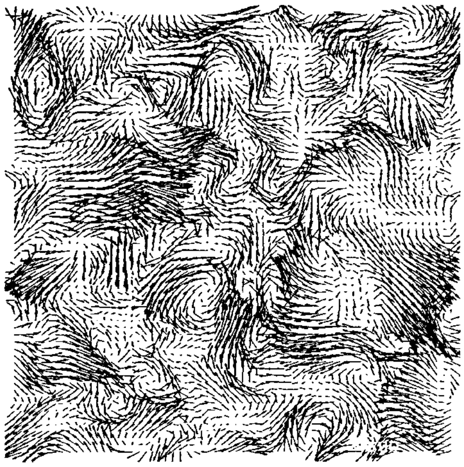

Figure 1. Turbulence with no imposed shear initially. Velocity field in a vertical section: initial condition.

2.5. Numerical Scheme

The Navier–Stokes equations are solved under the Boussinesq approximation. The method is called "pseudo-spectral" because the variables are the Fourier transforms of the physical fields but the non-linear terms are calculated in the physical space. This demands the use of a Fast Fourier Transform algorithm. The iteration scheme, called "slaved frog" [4] is of the second order in time and is unconditionally stable in the absence of non-linear terms.

The validation of the code has been performed in the case of decaying non-stratified turbulence. In these simulations, the turbulent kinetic energy was found to follow a power-law in time (not shown), with exponent very much in agreement with the experimental results of Corrsin [3].

3. Turbulence Decay

Decay of turbulence was simulated according to the previously described methodology. The three-dimensional flow is simulated with a 64^3 resolution, which allows $\mathrm{Re} = 50$ at $t = 0$. The Prandtl number is taken equal to unity and we choose the low value $\mathrm{Fr} = 0.1$ at $t = 0$ in order to highlight the properties of strongly stratified turbulence.

We show the velocity fields in a vertical section at $t = 0$ (Figure 1) and at the end of the decay, in the stratified and non-stratified cases (Figure 3). Figure 2 shows the evolution of the mean shear energy in the non-stratified and stratified cases. In order to use a constant and unique timescale for all graphs, we take the initial turnover timescale $T_t = l/U_{\mathrm{ref}}$ as the time unit.

First, we noticed alternative exchanges between poloidal and potential components (not shown) on short periods close to half the Brünt–Väisälä period. This

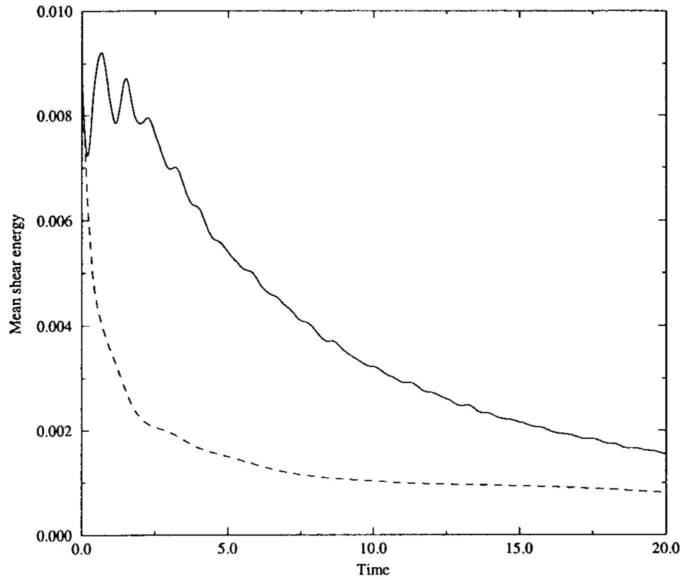

Figure 2. Turbulence with no imposed shear initially. Evolution of the mean shear energy, normalized by the initial total energy. ———: non-stratified case. —–: stratified case (Fr = 0.1). The time unit is the initial turnover timescale.

result is in agreement with other numerical studies (see, for example, [11]) and experimental results (such as those of Yoon et al. [16]). Hanazaki and Hunt [7] have recently shown that these oscillations could be theoretically explained by a

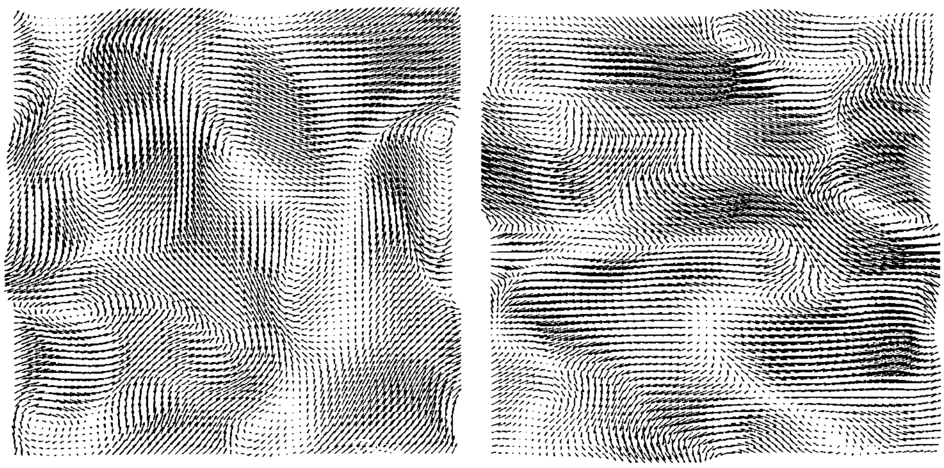

Figure 3. Turbulence with no imposed shear initially. Velocity fields in a vertical section. Left: end of the decay ($t = 20T_t$) in the non-stratified case. Right: end of the decay ($t = 20T_t$) in the stratified case (Fr = 0.1).

linear theory (Rapid Distortion Theory). Their results are confirmed in the case of a zero initial potential energy.

In this paper it is not our intention to further describe this oscillatory phenomenon, but we focus our attention on the evolution of the mean shear energy. Indeed, whereas this energy is monotonically dissipated in the non-stratified case, production is observed in the strongly stratified situation at the very beginning of the simulation. One will notice the low value of this shear energy compared to the other components of kinetic energy. Actually, the initial value of the mean velocity in a horizontal plane may follow a normal law since it is defined as the average of a great but finite number of random variables.

Furthermore, it is clear that the one-dimensional feature of the mean shear energy participates to the low value of its rate of viscous decrease since it is not concerned by the energetic cascade to dissipative small scales. However, this low dissipative feature cannot account for the production observed at the beginning of the decay.

In short, we observe that the mean shear energy, even if representing a small part of the total energy, is produced at the beginning of the decay with a rate higher than its (low) dissipation rate.

4. Turbulence Decay with an Initial Mean Flow

Simulation of decaying stratified turbulence with an imposed shear profile was performed in order to approach the velocity field configuration in the wake of a body moving in a stratified medium.

The initial condition is here the superposition of the initial condition used in Section 3 and a mean flow component given by:

$$V(z) = V_0 \cos\left(2\pi \frac{z}{L_b}\right),$$

where L_b is the size of the periodic box. V_0 is such that the mean flow energy is initially of the same order as poloidal and toroidal energies.

The simulation is still defined by the dimensionless numbers Re $= 50$ at $t = 0$, Pr $= 1$ and Fr $= 0.1$ at $t = 0$ in the stratified case.

We show the velocity fields in a vertical section at $t = 0$ (Figure 4) and at the end of the decay, in the stratified and non-stratified cases (Figure 7). Figure 5 shows the evolution of the mean shear energy in the non-stratified and stratified cases. The plots shown in Figure 6 are the evolution of energies divided by the instantaneous total energy, in the non-stratified and stratified cases. Thus, they give information on the partition of the total energy at each time step.

In the non-stratified case, the relative importance of the shear energy increases at the beginning of the decay. We attribute this phenomenon to the low dissipative feature of the large scale monodimensional mean flow compared to the other energies. However, after a few turnover time scales, the horizontal mean flow loses its

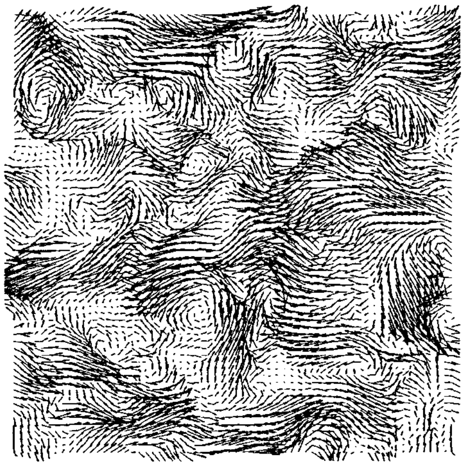

Figure 4. Turbulence with an initially imposed shear. Velocity field in a vertical section: initial condition.

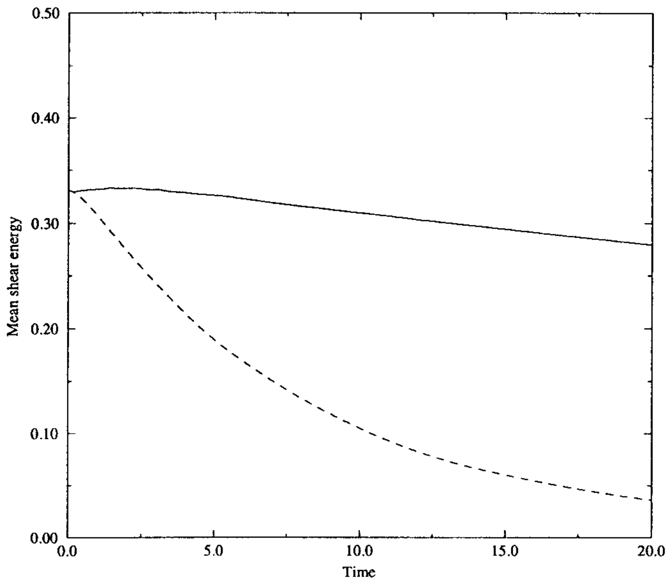

Figure 5. Turbulence with an initially imposed shear. Evolution of the mean shear energy, normalized by the initial total energy. – – –: non-stratified case. —-: stratified case (Fr $= 0.1$). The time unit is the initial turnover timescale.

coherence and its energetic relative importance decreases. At large t, no signature of the initial mean flow is visible and the velocity field is roughly isotropic.

On the contrary, in the stratified case, it is clear that the mean flow not only keeps its coherence but also is the dominating component of the flow at the end of the decay.

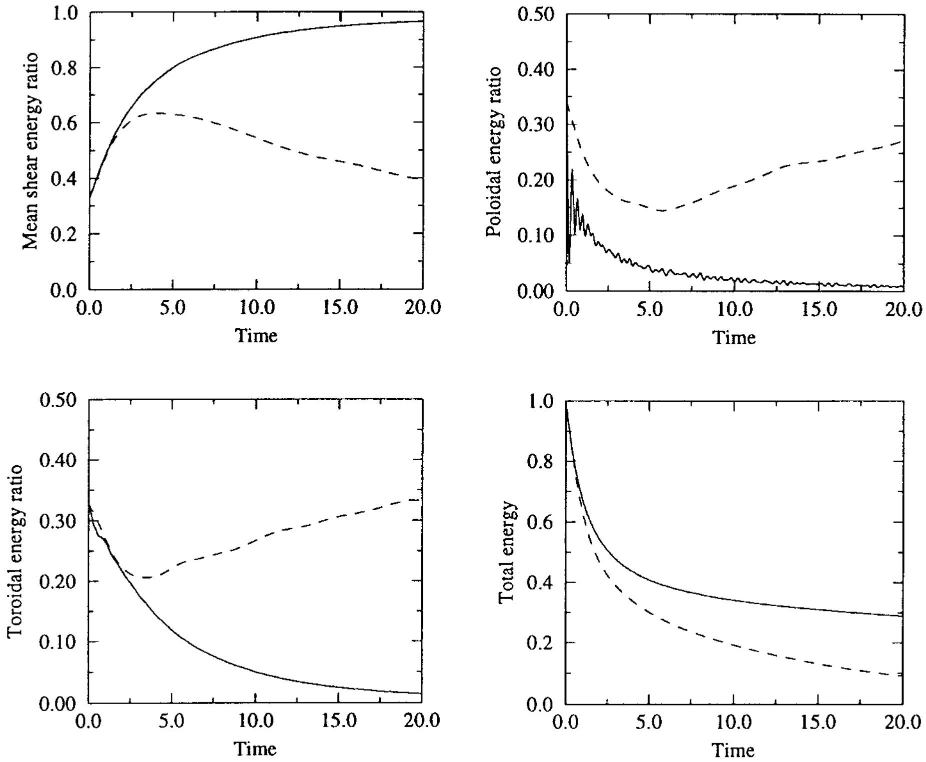

Figure 6. Turbulence with an initially imposed shear. Evolution of shear, poloidal and toroidal energies divided by the instantaneous total energy. — — —: non-stratified case. —–: stratified case (Fr = 0.1). The time unit is the initial turnover timescale.

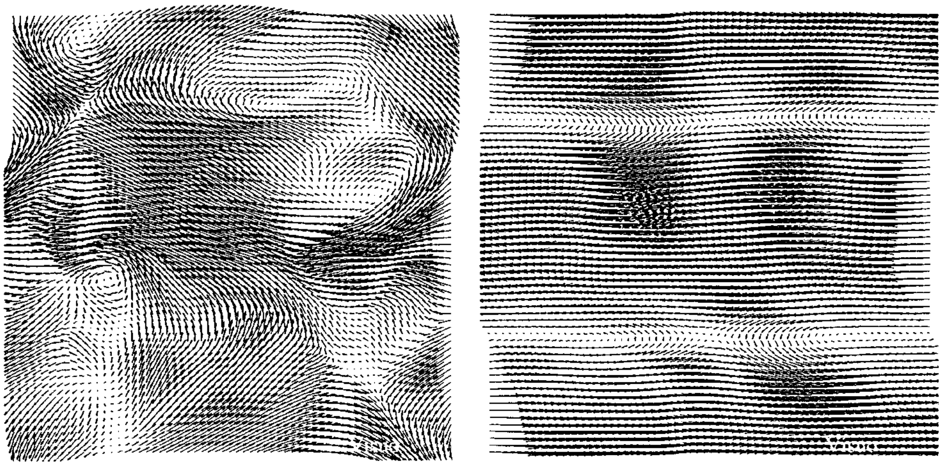

Figure 7. Turbulence with an initially imposed shear. Velocity fields in a vertical section. Left: end of the decay ($t = 20T_t$) in the non-stratified case. Right: end of the decay ($t = 20T_t$) in the stratified case (Fr = 0.1).

In the next section, we focus on wave-shear interaction which may be one possible shear production mechanism in stratified turbulent flows.

5. Analytical Elements on Wave-Shear Interaction

Turbulent motions in a stably stratified medium generate a gravity wave field which clearly has a great influence on the flow dynamics and, in particular, on mean motions. Our purpose in this section is to study the behaviour of one isolated internal wave in a horizontal parallel flow.

Propagation of internal waves in the presence of a mean horizontal flow was already theoretically and experimentally studied. Booker and Bretherton [1] analytically showed the possible existence of critical layers in such flows. Koop [9] experimentally studied the internal wave field generated by an oscillating cylinder in a non-uniform horizontal mean flow.

In this section, we give some elements showing that wave refraction by the mean horizontal flow can explain a spontaneous growth of this mean flow and possibly lead to a critical layer situation. One could introduce this phenomenon by invoking the Taylor–Goldstein equation and its singularities (as done by Koop [9], for instance). However, it is possible to get the same singularities by a simple refraction calculation with WKB hypothesis.

Let us consider a two-dimensional velocity field in the (y, z) plane corresponding to the refraction of a wave by a mean horizontal flow V(z,t). It will be assumed that $V(z, t)$ depends on z on a lengthscale which is much greater than the vertical wavelength of the internal wave. We would like to study the evolution of this coupled system, supposing that at each time the wave pattern is instantaneously refracted by the mean flow in the frame of the WKB theory.

Let $\phi(y, z, t)$ be the phase function of this refracted wave, with wave vector $\mathbf{k} = (k_2, k_3) = \nabla\phi$ and constant frequency $\omega = \partial_t\phi$. WKB assumptions provide invariance along the horizontal direction so that

$$\frac{\partial k_2}{\partial y} = \frac{\partial k_3}{\partial y} = 0.$$

Given that $\mathbf{k}$ is irrotational, this implies that:

$$\frac{\partial k_2}{\partial z} = \frac{\partial k_3}{\partial y} = 0.$$

Then, the wave field can be written as:

$$v = -\frac{k_3(z, t)}{k_2}\, w_0 \cos(k_2 y + \varphi(z, t) - \omega t), \tag{8}$$

$$w = w_0 \cos(k_2 y + \varphi(z, t) - \omega t), \tag{9}$$

where w_0 is the amplitude of the wave and $\partial_z\varphi = k_3(z, t)$.

Furthermore, let us write the Eikonal equation associated to the refracted wave:

$$\omega = k_2 V(z,t) + N \frac{k_2}{\sqrt{{k_2}^2 + k_3(z,t)^2}} ,$$

where N is the Brünt–Väisälä frequency.

On the other hand, the dimensional equation of the mean horizontal velocity $V(z,t)$ reads:

$$\frac{\partial V}{\partial t} + \frac{\partial}{\partial z} \langle vw \rangle^{xy} = \nu \frac{\partial^2 V}{\partial z^2}, \tag{10}$$

where $\langle \cdot \rangle^{xy}$ stands for the average on a horizontal plane.

We can calculate the shear production term $\partial_z \langle vw \rangle^{xy}$ due to the wave field:

$$\frac{\partial}{\partial z} \langle vw \rangle^{xy} = -\frac{\partial}{\partial z} \frac{{w_0}^2}{2} \frac{k_3(z,t)}{k_2} = -\frac{{w_0}^2}{2} \frac{\partial}{\partial z} \tan\alpha(z,t),$$

where $\alpha(z,t)$ is the angle between the wave vector and the horizontal direction.

Using the Eikonal equation, the expression of $tan\alpha$ is given by:

$$\tan\alpha = \sqrt{\frac{N^2}{[\omega - k_2 V(z,t)]^2} - 1}$$

and the evolution equation of $V(z,t)$ reads:

$$\frac{\partial V(z,t)}{\partial t} = \frac{{w_0}^2}{2} \frac{\partial}{\partial z} \sqrt{\frac{N^2}{[\omega - k_2 V(z,t)]^2} - 1} + \nu \frac{\partial^2 V(z,t)}{\partial z^2} , \tag{11}$$

or:

$$\begin{aligned} \frac{\partial V(z,t)}{\partial t} &= \frac{{w_0}^2}{2} \frac{N^2 k_2}{[\omega - k_2 V(z,t)]^2 \sqrt{N^2 - [\omega - k_2 V(z,t)]^2}} \\ &\times \frac{\partial V(z,t)}{\partial z} + \nu \frac{\partial^2 V(z,t)}{\partial z^2} . \end{aligned} \tag{12}$$

This equation is not easily solved in the general case. However, it allows us to show that the shear production term is singular when appear altitudes, z_{c1} where $\omega = k_2\ V(z_{c1},t)$ and z_{c2} where $\omega - k_2\ V(z_{c2},t) = \pm N$.

The first case accounts for the appearance of a critical layer. However, it is important to notice that a local increase (or decrease) of the mean horizontal velocity may happen even in the absence of a critical layer. Indeed, wave refraction contributes to the production term provided that $\partial_z \alpha$ is different from zero.

In the second case, we have $\tan\alpha = 0$. This corresponds to a horizontally propagating wave. Such a wave is thus able to transfer energy to the horizontal mean flow.

In the previous simulations, in which turbulence-induced waves and a mean flow coexist, the evolution of the mean shear energy may be explained in these terms. It is not easy to highlight the appearance of critical layers in turbulent situations. Nevertheless, in the next section we give a simple example showing a mean flow production by wave refraction and it may be that one can detect such a mechanism in the inner of fully developed turbulent flows.

6. Numerical Simulation of Wave-Shear Interaction

We have computed the decay of a stratified flow where the same horizontal mean flow, as in Section 4, initially coexists with a monochromatic internal wave. The initial Froude number is still Fr $= 0.1$ and Pr $= 1$. We use the same code as in the previous sections (see Sections 2.1, 2.2, 2.3 and 2.5): the Navier–Stokes equations are solved from a given initial condition and we observe the decay of the flow under the influence of viscous dissipation. Here, the initial condition is the superposition of the mean flow and a monochromatic gravity wave mode. The (two-dimensional) initial dimensional velocity field and the corresponding temperature field are given by:

$$u = 0, \tag{13}$$

$$v = -w_0 \cos(k_2 y + k_3 z) + V_0 \cos\left(2\pi \frac{z}{L_b}\right), \tag{14}$$

$$w = w_0 \cos(k_2 y + k_3 z), \tag{15}$$

$$\theta = \frac{N}{\alpha g} w_0 \sin(k_2 y + k_3 z). \tag{16}$$

We choose $k_2 = k_3$ so that $\mathbf{k} = (0, k_2, k_3)$ makes an angle of 45° with the horizontal direction. Furthermore, the wavelength is taken equal to one-fourth of the periodic box (which is of the order of the initial integral lengthscale l of the random velocity field previously simulated).

The wave amplitude w_0 and the mean motion amplitude V_0 are chosen such that the mean flow energy is initially equal to the poloidal energy. This induces an equipartition between poloidal, potential and mean shear energies at $t = 0$.

We present in Fig. 8 the velocity field in a vertical plane at $t = 0$ and $t = 4.5\ T_w$, where T_w is the Brünt–Väisälä period $2\pi/N$. Evolution of shear and total energies are shown on Figure 9.

Again, the main result we would like to focus on is the evolution of the mean shear energy. At $t \simeq 2T_w$, after a short decay, this energy begins to increase and at $t \simeq 8T_w$, the flow is almost totally dominated by the mean horizontal motion. It is clear from these observations that an energy transfer occurred from the internal wave mode to the mean flow mode. Furthermore, the low dissipative feature of this monodimensional flow induces a very slow decay of the total energy at large t.

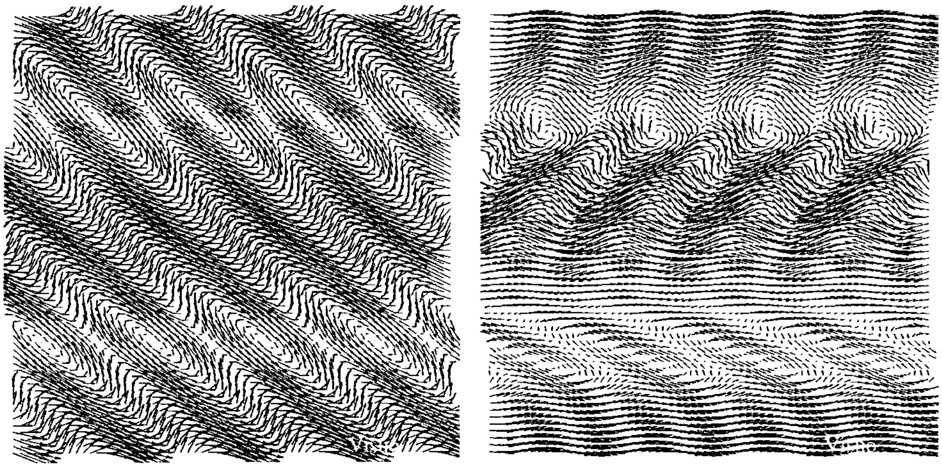

Figure 8. Wave-mean flow interaction simulation. Velocity fields in a vertical section. Left: $t = 0$. Right: $t = 4.5T_w$.

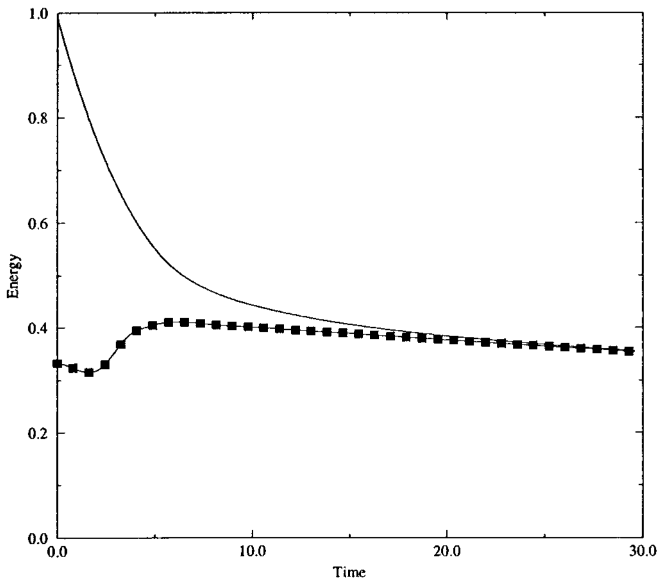

Figure 9. Wave-mean flow interaction simulation. Evolution of shear (■ – ■) and total (—-) energies, normalized by the initial total energy. The time unit is the Brünt–Väisälä period T_w.

7. Conclusion

The goal of this study was to focus on the behaviour of mean horizontal motions within developed turbulent flows and to give some elements on one possible shear production mechanism, say wave-shear interaction and critical layers. We have first simulated the decay of turbulence under the influence of a strong density gradient (Fr $= 0.1$ at $t = 0$) and plotted not only poloidal and toroidal energy evolutions but also the horizontal mean shear energy evolution. We showed that the latter, even if representing a small part of the total energy, is produced at the beginning of the decay and then slowly dissipated. We also performed calculations using the same initial conditions but adding a horizontal mean flow such as those encountered in turbulent wakes. The mean shear component is found to be the main component remaining at the end of the decay, since it is produced at the beginning and then slowly dissipated. We attributed the shear production phenomenon to the interaction between internal gravity waves and the mean flow itself. This may be a fundamental feature of stratified flows compared to non-stratified flows. To highlight this phenomenon, we gave some analytical elements on refraction of internal waves by a mean flow and showed that wave deviation participates to the production term of the mean flow. This production can lead to critical layer situations and may be one of the mechanisms responsible for the growth of the mean shear energy in the previously simulated turbulent stratified shear flows. As an illustration, we performed a simulation using as initial conditions a monochromatic internal wave coexisting with a horizontal mean flow. It was found that rapidly, all the energy was transferred to the mean flow component. This opens the way to further understand interactions between turbulence, internal gravity waves and mean motions.

To conclude, let us emphasize the importance that this phenomenon may have in environmental flows. Whereas in our simulations the damping effect of viscosity rapidly dissipates the energy, wave refraction and critical layers should fully develop in high-Reynolds number flows such as mixing layers or turbulent wakes in stratified geofluids.

Acknowledgements

We are grateful to M. Menneguzi for lending us his Gaussian velocity field generation code. This study was sponsored by the DRET and computing resources were supplied by the IDRIS.

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